IMPact
Volume 7(3)
University of Lincoln

Non-standard Quantum-like Models of Human Cognition

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Abstract

Human cognition, which refers to "the mental action or process of acquiring knowledge and understanding through thought, experience, and the senses" (Cambridge Cognition, 2015) is still a generally poorly understood subject among scientists and philosophers. In this article I reflect on the research carried out during my UROS summer 2023 project on mathematical models of human cognition. The main focus of the project was investigating a bistable optical illusion known as a Lissajous curve, which may be perceived to change directions when rotated at certain speeds, using quantum probability theory and a 'quantum Zeno model'. The research also considered existing studies of a conjunction fallacy, a famous error in human decision making, with both classical and quantum probability models, and a classic example of this known as the 'Linda problem'. The research and findings in this project form a good basis for further research and experiments on the subject.

Keywords and definitions:

- ***Lissajous curve****: a curve made up of sinusoidal waves overlaid on the same graph. It is named after Jules Antoine Lissajous who studied them in the 1800s* (Cadence PCB solutions, 2023)*.*
- ***Bistable perception****: a type of optical illusion in which one of two stable states can be perceived at any time. For example, the "duck or rabbit" optical illusion.*
- ***Quantum Zeno effect****: a phenomenon in quantum physics, where measuring a system twice in quick succession stops or slows down its time evolution. "A watched pot never boils" illustrates this.*
- ***Hilbert space****: a special type of vector space with some additional properties, important in quantum mechanics.*
- ***Quantum probability theory (QPT)****: a different approach to classical probability theory, in which possibilities of events are considered geometrically as subspaces within a Hilbert space* (Busemeyer, et al., 2014)*. Additionally in QPT, states can be considered to exist in superposition.*

Introduction

The project investigates some of the mathematical ‘quantum-like’ theories that may be used to model human cognition. Human cognition is studied in the form of optical illusions – in this case, a looping pattern of curves known as a Lissajous curve, which, when rotated, may appear to change its direction of rotation. This form of optical illusion is known as a bistable perception – since one of two different states of the image may be observed at any time. If the rotation of the Lissajous curve is fast enough, it may not appear to change direction at all. However, at a slower speed, the curve may appear to flip between clockwise and anticlockwise rotation.

One of the main theories that could explain this effect is the quantum Zeno effect; the idea that a particle that is constantly observed will never decay (Jones, 2019). The same idea can be considered in the context of this study – if measurements are taken frequently enough, and thus there is very little time between successive measurements, then perhaps the ‘flipping’ of the curve can be prevented, even for prolonged periods of time. In this context, rotation of the Lissajous curve at an optimal speed to allow for sufficient observations of the illusion and time to mentally process these measurements. This would make it very unlikely for the switch in direction to occur.

Project Background

In my third year of my undergraduate degree, I undertook the project “Mathematical Framework of Quantum Entanglement” which resulted in my bachelor's dissertation. I specifically requested the project due to my personal interest in quantum mechanics; which although I would have liked to, I had never had the opportunity to study on my university course since I study mathematics, and the module was only available to physics students. During the project, I found that I thoroughly enjoyed researching and writing and I took a particular interest in the real-life applications of the study. For example, quantum computing was one of these applications. My only wish was that I had more time to dedicate to the project without other coursework to distract me.

My supervisor suggested I could undertake a project that uses the same mathematics as the one I had studied, but in a different context – that the mathematics that explains quantum mechanics may also be able to model human cognition, and that this also coincided with his own research interests. Shortly after, I applied for the UROS project so that I would have the opportunity to research during the summer break without interruption, knowing that it would build on my existing knowledge of the subject, introduce some new concepts from another area, and prepare me well for starting my master’s year project.

Literature Review

A study by Boyer-Kassem, Duchêne and Guerci (Boyer-Kassem, et al., 2016) on the conjunction fallacy found that quantum-like models were not able to account for this effect. This study was one of the main motivations of my project.

The conjunction fallacy is the name given when humans judge that the conjunction of two events occurring (when the two events occur at the same time) is more likely than one of the individual events occurring; this violates the laws of classical probability. A classic example of this is the ‘Linda problem’ – the idea of a hypothetical, 31-year-old woman known as Linda who is “single, outspoken and very bright”. Linda is said to have “majored in philosophy, and as a student, she was deeply concerned with issues of discrimination and social justice and participated in anti-nuclear demonstrations”. Most participants judged that it was more likely for Linda to be both a bank teller and a feminist than just a bank teller alone. However, this cannot be the case according to classical probability theory:

If $P(B)$ denotes the probability that Linda is a bank teller, and $P(F)$ denotes the probability that Linda is a feminist, then classically we must have that

$$P(B \cap F) \leq P(B)$$

where $P(B \cap F)$ denotes the probability that Linda is both a bank teller and a feminist. This creates the conjunction fallacy, as according to this rule, the conjunction cannot be more probable than one of the individual components.

One explanation that has been suggested for this occurrence is to use quantum probability theory rather than classical to study human cognition, which is a more general framework that uses the mathematical tools of quantum mechanics. The study assesses this theory and concludes that it does not hold for the case of the conjunction fallacy. It does, however, suggest that ideas of quantum probability theory could be applied to other aspects of human cognition, including the disjunction fallacy or the inverse fallacy. The disjunction fallacy is explored a little more in the study on quantum cognition by Busemeyer and Pothos (Busemeyer & Pothos, 2022).

Methodology

Prior to beginning the research, I reviewed everything that I had done for my third-year project on the mathematics of quantum mechanics, focusing on specific relevant areas such as projection operators and Hilbert spaces. I also reflected on some of the things I learned from other relevant modules in previous years, including probability theory. Using the Lissajous curve example as a model for human cognition and perception, my goal was to find a suitable mathematical model that may be able to explain the bizarre flipping of the curve when it was rotated at a certain speed. As the project progressed, it became apparent that there would be a 'critical speed' for the flipping of the curve to occur, and that the rate at which the flipping occurred would depend on the rotation speed.

At the beginning of the project, I spent a lot of time searching for and reading through research papers on the subject due to the project being based on a very new and niche area of research. One of the papers that I focused on, *Quantum-like models cannot account for the conjunction fallacy* (Boyer-Kassem, et al., 2016), argued against the theory of quantum probability theory to explain this. I attempted to deconstruct part of the argument in this paper which referred to degenerate and non-degenerate systems, as it was unclear which one represented the case of the conjunction fallacy. This refers to the dimension of the eigenspace (non-degenerate eigenvalues have one-dimensional eigenspaces, whilst degenerate ones are multi-dimensional) and I thought this could relate to the complexity of a cognitive situation in which a human has to make a decision – the more complex ones would come with more options. For example, the aforementioned Linda problem would be a non-degenerate case. However, I was unable to make any firm conclusions on this theory and continued searching for other explanations. I spent the remainder of the project researching and testing the quantum Zeno effect model to see how it compared to the approximations.

Discussion and Results

So far in this research, I have found that the quantum Zeno effect model appears to give the best explanation of the perceived 'flipping' of the rotating Lissajous curve. Using classical probability theory, the decay of a bistable perception can be modelled similarly to the decay of radioactive material; by the exponential decay law:

$$N(t) = N_0 e^{-\Gamma t}$$

where Γ is the rate of rotation, t refers to time, N_0 is the initial number of subjects and $N(t)$ is the number of subjects that still perceive the curve to be rotating in the same direction that it was initially after a time t has passed. The 'survival probability' (the probability that a person's perception has not changed, and so the curve has not changed direction) in this case can be approximated to be:

$$S_c(t) \approx 1 - \Gamma t + \cdots$$

for small values of t only. The classical model can be considered as an approximation only and is not reliable when considering measurements made over extended periods of time. It also relies on classical probability theory only. This introduces the quantum Zeno model, which makes use of quantum probability theory.

In the quantum model, the survival probability decays much more slowly due to the way it is derived. It decays quadratically rather than linearly as in the classical case, and for small values of t, it can be approximated as:

$$S_q(t) \approx 1 - \Gamma^2 t^2 + \cdots$$

Again, this is unreliable for large values of t, when measurements are taken over an extended period. When t is not small, the model for survival probability is much more complex and predicts that the average lifetime of the bistable perception is revived once decayed, and that the cycle repeats. I will be taking part in further research on this subject and so have not included the specifics of this model, however the following plot generated in Python illustrates how the average lifetime of the bistable perception is revived in comparison to the approximation for small values of t:

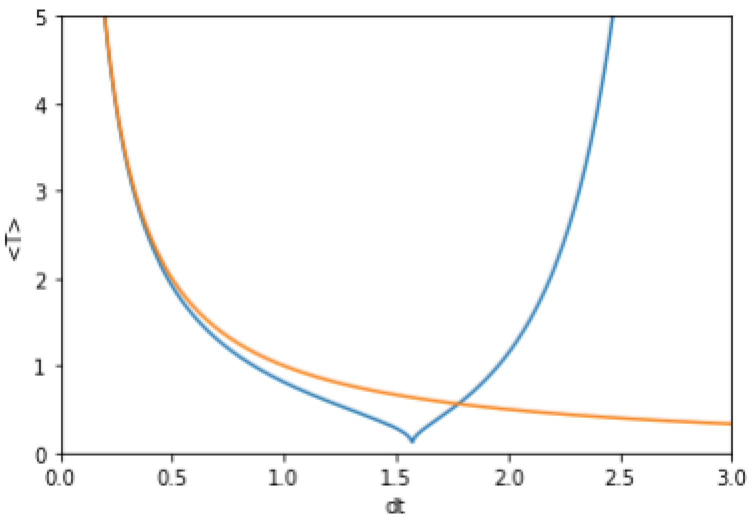

Figure 1: plot generated in Python. The orange line represents approximation for small values of t, the blue line represents quantum Zeno model of bistable perception for all values of t.

Here, the horizontal axis, dt, represents the time in between measurements. The vertical axis, $< T >$, represents the lifetime of a perception state; how long the perception is of the curve rotating clockwise or anticlockwise lasts before switching to the opposite direction.

UROS Experience

The UROS experience has given me a most valuable insight into academic and research career opportunities. It has also been an overall positive and unique experience for me as I have never previously had to come up with original research ideas, which was a large part of this project. In my opinion this part of the project was also the most challenging – it was easy to spend lots of time looking into potential ideas and theories and lose focus on the main ideas of the project, especially if those ideas were not found to be of much relevance to the original research problem.

I have thoroughly enjoyed having the chance to collaborate with academics within my school throughout the summer to work on a new area of research and they have been extremely helpful in their support and guidance. The experience has helped to develop my knowledge of the research area, which besides being of personal interest will be greatly relevant and beneficial to my master's year project, in which I plan to continue research in the same field with the aim of obtaining complete results and answers to the proposed questions. Additionally, it has helped to build on my existing research experience as an undergraduate student and has given me more confidence in going into my master's year. I would greatly recommend undertaking a UROS project to anyone considering studying a PhD or a career in academic research, or students looking to add to their research experience.

Conclusion

To conclude, the quantum Zeno model was found to be the most reliable model from the ones investigated to be able to explain a bistable perception in the Lissajous curve example. I intend to continue research in this area alongside my supervisors and hope to test the theory using eye-tracking experiments to see if the actual data agrees with the model. I would also like to investigate examples in addition to the rotating Lissajous curve; for example auditory illusions, to see if the same results can be obtained, which would further support the quantum Zeno effect model for human cognition. Additionally, I think that quantum-like models using degeneracy and non-degeneracy could be investigated further to try to explain the disjunction or inverse fallacy, or other aspects of human cognition, and this could form part of a future research project.

This research has some useful theoretical and practical implications; suggesting a potentially suitable model for some aspects of human cognition. This may be extended to be used in predicting human behaviour or other cognitive patterns and improving scientific understanding of human cognition.

Overall, I have found the UROS experience to be an extremely positive one and I am grateful to have been provided with this fantastic opportunity.

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